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On the propagation of sound in low-dimensional conducting structures

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Abstract. Attenuation of sound wave energy in layered conductors with several charge-carrier groups is investigated theoretically. The existence of a group with a quasi-one-dimensional energy spectrum is shown to affect the behaviour of the rate of sound attenuation G in a magnetic field $\boldsymbol{H}$ fundamentally. Giant oscillations, as well as resonant peaks of G depending on $1/H$, are predicted.

1. Introduction

The search for new superconducting materials has focused attention on conductors of organic origin that possess layered structure. Due to their specific behaviour these conductors have become objects of intensive experimental study both in the normal and superconducting states. In this connection the analysis of their electronic structure seems to be useful.

Many organic layered conductors have metal-type electrical conductivity, and the well-developed concept of quasi-particles carrying a charge in metals can be applied for describing their electronic properties.

At the temperature of liquid helium, the Shubnikov–de Haas effect is clearly manifested in the organic conductors, which proves that the samples used in experiments are mono-crystals with the charge-carrier free-path length significantly exceeding the radius of curvature of the electron trajectory in a strong magnetic field. Under these conditions the electron energy spectrum of layered conductors can be studied by means of measurements of their kinetic characteristics. In particular, acoustic phenomena should prove informative and useful for investigating the electronic structure in detail.

Organic conductors show sharp anisotropy in their electrical conductivity: the conductivity along the layers is substantially higher than that along the normal to the layers. This is apparently connected with the anisotropy of the velocities on the Fermi surface and restricts the choice of suitable models for the Fermi surface. The form of a weakly warped cylinder is in good agreement with the experimental studies of galvanomagnetic phenomena and the Shubnikov–de Haas effect in salts of tetrathiafulvalene of the type $(BEDT\text{-}TTF)_2JBr_2$ [1–5]. However, the unusual behaviour of the magnetoresistance of the family of salts $(BEDT\text{-}TTF)_2MHg(SCN)_4$ (M = K, Rb, Te) [6, 7] suggests that the Fermi surface of such layered conductors is complicated and may contain two quasi-1D sheets in addition to a weakly warped cylinder. The sheets are weakly warped planes on which the velocity of charge carriers has a preferred direction in the layer plane.

2. Theoretical model

We consider the propagation of the acoustic waves in low-dimensional conductors placed in an external magnetic field $\boldsymbol{H}$. The sound wave can be described by means of the elasticity theory equation for the ionic displacement $\boldsymbol{u}$:

$$-\omega^2 \rho u_i = \lambda_{ijlm} \frac{\partial u_{lm}}{\partial x_j} + F_i \tag{1}$$

where ρ and λ_{ijlm} are the density and elastic tensor of the crystal, $u_{lm} = (\partial u_l/\partial x_m + \partial u_m/\partial x_l)/2$ is the deformation tensor. The acoustic wave is taken to be monochromatic with frequency ω, so the differentiation with respect to the time variable is equivalent to multiplication by $(-\mathrm{i}\omega)$. Perturbation of the electron system due to the sound causes the appearance of the force

$$F_i = \mu_0 (\boldsymbol{j} \times \boldsymbol{H})_i + \frac{\mathrm{i} m\omega}{e} j_i + f_i^d \tag{2}$$

acting upon the lattice from the conduction electrons [8, 9]. Here m and e are the electron mass and charge, μ_0 is the magnetic permeability in vacuum.

The electric current density

$$j_i = -\frac{2}{(2\pi\hbar)^3} \int e v_i \psi \frac{\partial f_0}{\partial \varepsilon}\, \mathrm{d}^3 p \equiv \langle e v_i \psi \rangle \tag{3}$$

and the deformation force density

$$f_i^d = \frac{\partial}{\partial x_k} \langle \Lambda_{ik} \psi \rangle \tag{4}$$

are determined by the solution of the kinetic equation for the charge-carrier distribution function $f_0\{\varepsilon(\boldsymbol{p}) + \mathrm{i}\omega \boldsymbol{p}\cdot\boldsymbol{u}\} - \psi\, \partial f_0/\partial\varepsilon$ in the concomitant coordinate system which moves together with the crystal lattice with the velocity $-\mathrm{i}\omega\boldsymbol{u}$.

Here f_0 is the Fermi distribution function, $\boldsymbol{v}$, $\boldsymbol{p}$ and $\varepsilon(\boldsymbol{p})$ are the electron velocity, momentum and energy, respectively; Λ_{ik} is determined by expressions (8) and (9).

The function ψ determines the non-equilibrium state of the electron system and can be found from the kinetic equation. In the linear approximation under weak perturbation of the electron system by the sound wave the kinetic equation takes the form

$$\begin{gathered} \boldsymbol{v} \frac{\partial \psi}{\partial \boldsymbol{r}} + \frac{\partial \psi}{\partial t} + \left(\frac{1}{\tau} - \mathrm{i}\omega \right) \psi = g \\ g = -\mathrm{i}\omega \Lambda_{ij}(\boldsymbol{p}) u_{ij} + e\tilde{\boldsymbol{E}} \cdot \boldsymbol{v}. \end{gathered} \tag{5}$$

The collision operator is represented by the approximation of the relaxation time τ. Time t determines the position of a charge on its trajectory in a magnetic field according to the equation of motion

$$\frac{\partial \boldsymbol{p}}{\partial t} = e\boldsymbol{v} \times \boldsymbol{H}. \tag{6}$$

In the concomitant coordinate system the perturbation of electrons by the sound wave is connected with the electric field

$$\tilde{\boldsymbol{E}} = \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{u} \times \boldsymbol{H} + \frac{m\omega^2 \boldsymbol{u}}{e} \tag{7}$$

and with the renormalization of the energy spectrum under the strain

$$\delta\varepsilon = \lambda_{ij}(\boldsymbol{p}) u_{ij}. \tag{8}$$

The kinetic equation contains the components of the deformation potential tensor $\lambda_{ij}(\boldsymbol{p})$ in the form that accounts for the conservation of the charge-carrier number, i.e.

$$\Lambda_{ik}(\boldsymbol{p}) = \lambda_{ik}(\boldsymbol{p}) - \frac{\langle \lambda_{ik}(\boldsymbol{p}) \rangle}{\langle 1 \rangle}. \tag{9}$$

The components of the electric field $\boldsymbol{E}$ generated by the sound wave should be determined from the Maxwell equation

$$\text{curl curl}\, \boldsymbol{E} = \mathrm{i}\omega\mu_0 \boldsymbol{j} \tag{10}$$

and the electroneutrality condition which reduces to the continuity condition for the electric current

$$\text{div}\, \boldsymbol{j} = 0. \tag{11}$$

Because of the high density of charge carriers in a conductor, the displacement current is neglected.

The solution of the kinetic equation can be expressed as

$$\psi = \int_{-\infty}^{t} \mathrm{d}t'\, g[x + x(t') - x(t)] \exp[\nu(t' - t)] \tag{12}$$

where $\nu = 1/\tau - \mathrm{i}\omega$.

Let the sound wave propagate along the x-axis. In the Fourier representation equations (1), (10) and (11) reduce to a set of equations for the Fourier components, ionic displacement $\boldsymbol{u}(\boldsymbol{k})$ and electric field $\tilde{\boldsymbol{E}}(k)$:

$$\begin{aligned}
&\mathrm{i}\omega\mu_0 j_\alpha(k) = k^2\big[\tilde{E}_\alpha(k) + \mathrm{i}\omega\mu_0\big(\boldsymbol{u}(k) \times \boldsymbol{H}\big)_\alpha\big] \qquad \alpha = y, z \\
&j_x(k) = 0 \\
&-\omega^2 \rho u_i(k) = -\lambda_{ixlx} k^2 u_l + (\mathrm{i}m\omega/e) j_i(k) + \mu_0\big(\boldsymbol{j}(k) \times \boldsymbol{H}\big)_i + \mathrm{i}k\langle \Lambda_{ix}\psi(x)\rangle.
\end{aligned} \tag{13}$$

Using the solution of the kinetic equation, we can express $j_i(k) = \langle e v_i \psi(k)\rangle$ and $\langle \Lambda_{ix}\psi(k)\rangle$ in the forms

$$\begin{aligned}
&j_i(k) = \sigma_{ij}(k)\tilde{E}_j(k) + a_{ij}(k)k\omega u_j(k) \\
&\langle \Lambda_{ix}\psi(k)\rangle = b_{ij}(k)\tilde{E}_j(k) + c_{ij}(k)k\omega u_j(k).
\end{aligned} \tag{14}$$

The Fourier transforms of the electrical conductivity tensor and acoustoelectronic tensors $a_{ij}(k)$, $b_{ij}(k)$, $c_{ij}(k)$ are described by the following expressions:

$$\begin{aligned}
&\sigma_{ij}(k) = \langle e^2 v_i \hat{R} v_j\rangle \qquad a_{ij}(k) = \langle e v_i \hat{R}\Lambda_{jx}\rangle \\
&b_{ij}(k) = \langle e\Lambda_{ix}\hat{R} v_j\rangle \qquad c_{ij}(k) = \langle \Lambda_{ix}\hat{R}\Lambda_{jx}\rangle
\end{aligned} \tag{15}$$

where

$$\hat{R}g \equiv \int_{-\infty}^{t} \mathrm{d}t'\, g(t') \exp\{\mathrm{i}k[x(t') - x(t)] + \nu(t' - t)\}.$$

By substituting equations (15) into the equation set (13), we obtain a set of algebraic equations which is linear with respect to $u_i(x)$ and $\tilde{E}_i(x)$. The condition for the existence of a non-trivial solution of this set of equations (equating the system determinant to zero) represents the dispersion equation for the problem. The imaginary part of the roots of the dispersion equation determines the attenuation of the acoustic and electromagnetic waves, and the real part describes the renormalization of their velocities. Because of the great mass difference of the ions and the electrons, the root k related to the sound wave and the root k_e related to the electromagnetic wave differ significantly.

To clarify the specific features of the propagation of sound throughout the low-dimensional conductors we consider a layered conductor with two groups of charge carriers: the quasi-1D group whose charge carriers obey an energy–momentum relation of the type

$$\varepsilon_1(\boldsymbol{p}) = \pm \boldsymbol{p} \cdot \boldsymbol{N} v + \eta_1 \frac{\hbar}{a} v \cos\left(\frac{a p_z}{\hbar}\right) \tag{16}$$

and the quasi-2D one with the dispersion law of the type

$$\varepsilon(\boldsymbol{p}) = \frac{p_x^2 + p_y^2}{2m} + \eta \frac{\hbar}{a} v_0 \cos\left(\frac{a p_z}{\hbar}\right) \qquad v_0 = \left(\frac{2\varepsilon_F}{m}\right)^{1/2}. \tag{17}$$

Here $\eta_1, \eta \ll 1$; a is the separation between layers, $\hbar$ is the Planck constant, v is the velocity of electrons with the Fermi energy ε_F belonging to the quasi-1D sheet of the Fermi surface. The unit vector $\boldsymbol{N} = \{\cos\phi, \sin\phi, 0\}$ lies in the plane of the layers and makes an angle ϕ with the direction of the wave propagation.

3. Calculation

Let us consider the longitudinal acoustic wave. Using equations (13)–(15), it is easily seen that for $\omega\tau \ll 1$ in the limit of small η and η_1, the root k of the dispersion equation can be represented as follows:

$$k = \frac{\omega}{s} + k_1 \tag{18}$$

where the small correction k_1 takes the form

$$k_1 = \frac{\mathrm{i}k^2}{2\rho s} \frac{1}{(1 - \xi\tilde{\sigma}_{yy})} \left\{ \xi(\tilde{a}_{yx}\tilde{b}_{xy} - \tilde{c}_{xx}\tilde{\sigma}_{yy}) + \tilde{c}_{xx} - \mathrm{i}(\tilde{a}_{yx} - \tilde{b}_{xy}) \frac{H_z \mu_0}{k} + \tilde{\sigma}_{yy} \frac{H_z^2 \mu_0^2}{k^2} \right\}\Bigg|_{k=\omega/s} \tag{19}$$

with $s = (\lambda_{xxxx}/\rho)^{1/2}$; $\xi = \mathrm{i}\omega\mu_0/k^2$ and

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \frac{\sigma_{\alpha x}\sigma_{x\beta}}{\sigma_{xx}} \qquad \tilde{a}_{\alpha j} = a_{\alpha j} - \frac{a_{xj}\sigma_{\alpha x}}{\sigma_{xx}}$$
$$\tilde{b}_{i\beta} = b_{i\beta} - \frac{b_{ix}\sigma_{x\beta}}{\sigma_{xx}} \qquad \tilde{c}_{ij} = c_{ij} - \frac{b_{ix}a_{xj}}{\sigma_{xx}} \tag{20}$$

for $\alpha, \beta = y, z$.

The integration in the expressions (15) must be taken over all of the sheets of the Fermi surface and each of the components is the sum of the contributions from the quasi-1D and quasi-2D groups of electrons.

If the magnetic field is not directed in the layer plane, the charge carriers of the quasi-2D group gyrate along closed orbits with the frequency Ω. In the range of the magnetic fields where the radius of curvature of the electron orbit is much less than the electron mean free path l but exceeds the sound wavelength $1/k$ significantly, the electrons of the quasi-2D group are involved in Pippard oscillations [10]. This effect is associated with the periodic repetition of the conditions for the most effective interaction between electrons and the sound wave. Under the Pippard effect conditions the contributions $\sigma_{ij}^{(2)}, a_{ij}^{(2)}, b_{ij}^{(2)}, c_{ij}^{(2)}$ to the acoustoelectronic tensors from electrons of the quasi-2D group can be easily calculated by the method of stationary phases. In doing so, the assumption is that components of the tensor $\Lambda_{ik}^{(2)}(\boldsymbol{p})$ coincide in order of magnitude with the Fermi energy and that the magnetic field is oriented along the z-axis.

We present the expressions for some of the components:

$$\sigma^{(2)}_{yy} = \frac{4N_2e^2}{m\nu\pi kD}[1 - J_0(\zeta)\sin kD]$$
$$\sigma^{(2)}_{yx} = -\sigma^{(2)}_{xy} = \frac{4N_2e^2}{m\nu\pi kDkl}J_0(\zeta)\cos kD \tag{21}$$
$$c^{(2)}_{xx} = \frac{N_2mv_0^2}{\pi\nu kD}[1 + J_0(\zeta)\sin kD].$$

Here and in what follows, $N_{1,2}$ are the densities of charge carriers of the quasi-1D and quasi-2D groups, respectively. $J_0(\zeta)$ is the Bessel function, $\zeta = kR\eta$, $R = 2\hbar/e\mu_0Ha$, $D = 2v_0m/e\mu_0H$.

The other components of the acoustoelectronic tensor behave in an analogous way. At $\zeta \ll 1$ almost all charge carriers with the quasi-2D energy spectrum contribute to the oscillations and, in contrast to the case for an ordinary metal, the amplitude of the oscillations is great.

The presence of the preferred direction for the velocities of the electrons of the quasi-1D group manifests itself in the dependence of their deformation potential $\Lambda^{(1)}_{ij}$ on ϕ. If deformation of the crystal does not cause redistribution of charges between the electron groups, it is reasonable to suppose, keeping in mind formula (9), that $\Lambda^{(1)}_{ij}$ vanishes in the zeroth approximation in the small parameter η_1. If we set $\Lambda^{(1)}_{xx} = -\eta_1\varepsilon_F\cos\phi$, the expressions for the acoustoelectronic coefficients take the form

$$\sigma^{(1)}_{\alpha\beta} = h_\phi\frac{N_1e^2v^2}{\nu\varepsilon_F}N_\alpha N_\beta \qquad \alpha,\beta = x,y \qquad c^{(1)}_{xx} = \eta_1^2h_\phi\frac{N_1\varepsilon_F\cos^2\phi}{\nu}$$
$$a^{(1)}_{xx} = b^{(1)}_{xx} = \mathrm{i}\eta_1h_\phi\frac{N_1ev}{\nu}kl\cos^3\phi \tag{22}$$
$$a^{(1)}_{yx} = b^{(1)}_{xy} = \mathrm{i}\eta_1h_\phi\frac{N_1ev}{\nu}kl\cos^2\phi\sin\phi$$

where

$$h_\phi = [1 + (kl)^2\cos^2\phi]^{-1} \qquad l = v\tau.$$

4. Results and discussion

In the main approximation in the small parameters $(\Omega\tau)^{-1}$ and $(kD)^{-1}$, the sound attenuation rate is

$$G = G_0\Omega\tau\frac{1 - J_0^2(\zeta) + kDg_\phi[1 + J_0(\zeta)\sin kD] + \eta_1^2kDf_\phi^2\cos^2\phi[1 - J_0(\zeta)\sin kD]}{1 - J_0(\zeta)\sin kD + kDg_\phi}\bigg|_{k=\omega/s} \tag{23}$$

where $G_0 = N_2m\omega v_0/4\pi\rho s^2$, $\Omega = e\mu_0H/m$; the functions

$$f_\phi = \frac{N_1}{N_2}\frac{(kl)^2\cos^2\phi}{1 + (kl)^2\cos^2\phi} \qquad g_\phi = \frac{N_1}{N_2}\frac{\sin^2\phi}{1 + (kl)^2\cos^2\phi}$$

do not exceed unity in the case where the densities of the charge carriers of the first and the second groups are equal. In expression (23) we have neglected unity in comparison with the magnitude $|\zeta\tilde{\sigma}_{yy}|$. This corresponds to the inequality $\pi^2\omega^2D/s^3\omega_0^2\tau\mu_0 \ll 1$, satisfied in the range of ultrasonic frequencies if the plasma frequency ω_0 is comparable to that of ordinary metals. Inessential numerical factors of the order of unity in formula (23) have been omitted.

The presence of the charge-carrier group with the quasi-1D dispersion law leads to the essential anisotropy in the layer plane of the sound wave attenuation. If the wave propagation is along the preferred direction of the velocities of the electrons belonging to this group ($\phi = 0$), the rate of sound wave attenuation can be represented as follows:

$$G = G_0 \left(\Omega\tau \frac{1 - J_0^2(\zeta)}{1 - J_0(\zeta)\sin kD} + \eta_1^2 \frac{N_1^2}{N_2^2} \frac{\omega\tau v_0}{s} \right) \bigg|_{k=\omega/s}. \tag{24}$$

At $\zeta \ll 1$ the corrugation of the quasi-2D sheet of the Fermi surface is small enough that the first term in formula (24) acquires the form of sharp resonant peaks. The resonant dependence of G on H^{-1} can be observed by measurements of the derivative of G with respect to the magnetic field. In this case charge carriers with the quasi-1D energy spectrum contribute to the 'background' part of G. The case where the 2D group exists alone ($N_1 \equiv 0$) leads to the result given in [11].

If the angle ϕ is being deflected from $\phi = 0$, then the resonant character of the $G(H^{-1})$ dependence holds as long as $\pi/2 - \phi > (kD)^{1/2}/kl$. When the angle ϕ approaches $\pi/2$, the resonant behaviour of the sound attenuation rate changes in the course of giant oscillations, which at $\phi = \pi/2$ take the form

$$G = G_0\Omega\tau\{1 + J_0(\zeta)\sin kD\} \simeq G_0\Omega\tau \left\{ 1 + \sin kD - \frac{\zeta^2}{4}\sin kD \right\} \bigg|_{k=\omega/s}. \tag{25}$$

At $\sin kD = -1$ the sound attenuation rate G attains its minimum value, which is less when the corrugation of the Fermi surface is weaker.

The numerical calculations based on formula (23) are analysed graphically in 3D space. The dependence of the acoustic absorption coefficient G/G_0 on the magnetic field $h = H_0/H$, where $H_0 = 2\omega v_0 m/es\mu_0$, and the cosine of the angle $x = \cos\phi$ at $\eta = \eta_1 = 10^{-2}$, $N_1/N_2 = 1$, $kl = 10^2$ is shown in figure 1. It is easy to see that under these conditions the inequalities $\omega\tau \ll 1$, $\Omega\tau \gg 1$, $\zeta \ll 1$, $1/k \ll r \ll l$, are valid.

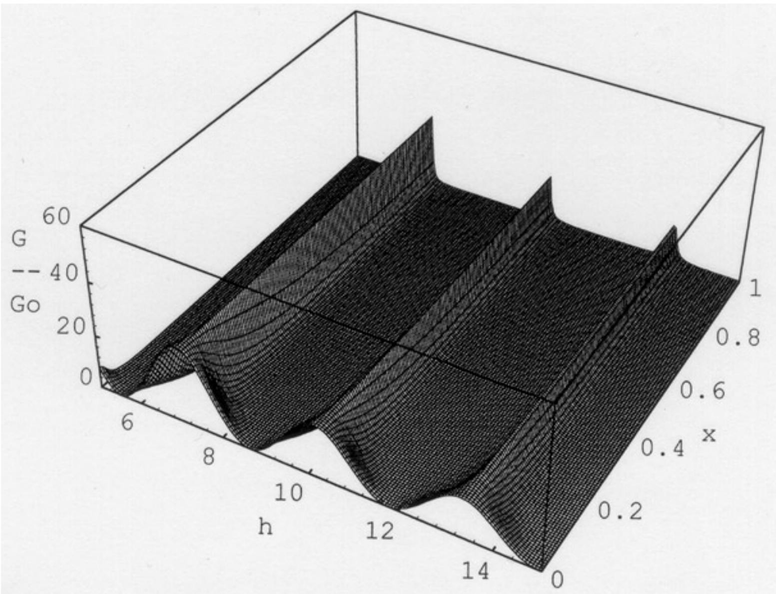

Figure 1. The dependence of the acoustic absorption coefficient G/G_0 on the magnetic field $h = H_0/H$ and the angle between the preferred direction of the 1D group of electrons $x = \cos\phi$ at $\eta = \eta_1 = 10^{-2}$, $kl = 10^2$, $N_1 = N_2$.

In the presence of the quasi-1D charge-carrier group, the resonance dependence of the rate of sound attenuation transforms into giant oscillations when the direction of the acoustic wave propagation changes.

The cross sections of the graphic (figure 1) in the planes $x = 0$ and $x = 1$ are presented in figures 2 and 3, respectively. The resonant peaks are given when the wave propagates parallel to the preferred direction of the velocities of the electrons belonging to the 1D group ($x = 1$; figure 3). If the wave propagates in the direction perpendicular to the preferred direction, the giant oscillations appear ($x = 0$; figure 2).

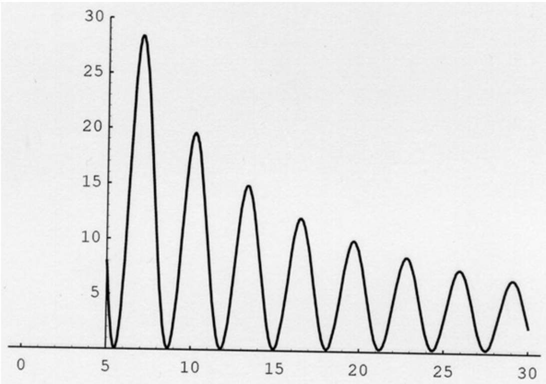

Figure 2. The dependence of the acoustic absorption coefficient G/G_0 on the magnetic field at $\eta = \eta_1 = 10^{-2}$, $kl = 10^2$, $N_1 = N_2$, $x = 0$.

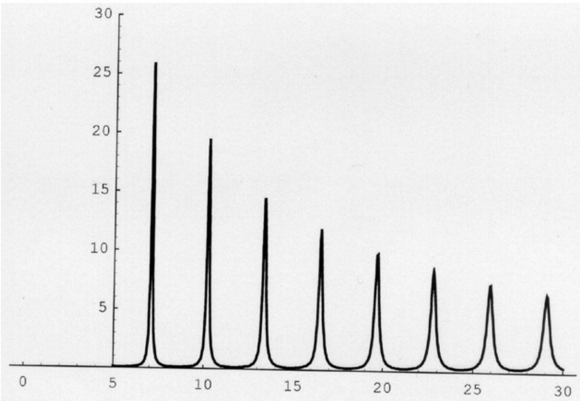

Figure 3. The dependence of the acoustic absorption coefficient G/G_0 on the magnetic field at $\eta = \eta_1 = 10^{-2}$, $kl = 10^2$, $N_1 = N_2$, $x = 1$.

Thus the observation of the giant oscillations of the rate of sound attenuation is proof of the existence of the quasi-1D group of carriers.

Magnetoacoustic measurements in layered conductors enable the preferred direction for velocities of charge carriers belonging to this group as well as the degree of low dimensionality of the electron energy spectrum to be determined.

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